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Spontaneous symmetry breaking and switching in planar nonlinear optical antiwaveguides

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We consider guided light beams in a nonlinear planar structure described by the nonlinear Schrödinger equation with a symmetric potential hill. Such an ''antiwaveguide'' (AWG) structure induces a transition from symmetric to asymmetric modes via a transcritical pitchfork bifurcation, provided that the beam's power exceeds a certain critical value. It is shown analytically that the asymmetric modes always satisfy the Vakhitov-Kolokolov (necessary) stability criterion; nevertheless, the application of a general Jones' theorem shows that the AWG modes are always unstable. To realize the actual character of the instability, we perform direct numerical simulations, which reveal that a deflecting instability, which drives the asymmetric beam into the cladding without giving rise to fanning or stripping of the beam, sets in after a propagation distance of approximately 16 transverse widths of the AWG's core. The symmetry-breaking bifurcation, in combination with the deflecting instability, may be used to design an all-optical switch. The switching can easily be controlled by means of a symmetry-breaking ''hot spot'' that acts upon an initial symmetric beam launched with a power exceeding the bifurcation value.

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I. INTRODUCTION

It is commonly known that the bound states in the one-dimensional linear Schrödinger equation with a symmetric potential well are always symmetric or antisymmetric. It may seem plausible that this is also true for nonlinear waveguides (WGs), which are described by the nonlinear Schrödinger (NLS) equation. In the present work, we demonstrate that, while this is indeed true for the NLS equation with a symmetric potential well, which describes the nonlinear WG proper, spontaneous symmetry breaking occurs, via a transcritical bifurcation, in the case when the NLS equation contains a potential hill, rather than a well. This configuration corresponds to a nonlinear *antiwaveguide* (AWG), i.e., a structure with the reverse refractive-index difference between the core and cladding.

In the linear approximation, i.e., when the optical power is small, the light is repelled by an AWG. The beam's power must exceed a certain threshold level for trapping by an AWG, the threshold power being of the same order of magnitude as that for self-focusing [1]. The symmetry-breaking bifurcation takes place when the power exceeds another (slightly larger) critical value.

A very important issue is the stability of modes trapped by the AWG. It might naturally be expected that they may never be completely stable. Below, we confirm this expectation, using a general theorem by Jones [2]. Our direct simulations demonstrate that, in some cases, the instability is slow, allowing for the propagation of AWG modes over a distance essentially exceeding the characteristic diffraction length. However, the actual purport of the study of AWG modes is not an attempt to make them less unstable, but, instead, their potential for use in all-optical switching devices, as proposed below. In fact, the moderate instability of the modes will be quite *useful* in this context, making it possible to control the switching efficiently and reduce the necessary size of the switch.

AWG-based switching includes two stages which, in fact, can be integrated together. In the first stage, an initial symmetric beam with a power exceeding the bifurcation-generating value is converted into an asymmetric beam under the action of an external controllable disturbance in the form of a small ''hot spot.'' The particular asymmetric state is chosen, out of the two mutually symmetric ones, by the location of the hot spot relative to the system's axis. In the second stage, the asymmetric beam develops a *deflecting instability*, which drives it into the cladding. The instability does not violate the coherent structure of the beam, giving rise to no conspicuous fanning or stripping. In fact, the same hot spot that controls the choice of the bifurcation branch plays the role of a push that initiates the development of the deflecting instability.

An advantage of AWGs for potential applications is that they have small cross sections of both the core and the trapped light beam. The cross-section size is near the physical limit, i.e., on the order of the wavelength (it is quite possible technologically to fabricate structures with cross-section size of this order of magnitude).

A characteristic feature of AWGs, which is not possible at all in WGs, is the existence of special values of the propagation constant, in very narrow vicinities of which the diffraction is balanced by the self-focusing in *finite intervals* of values of the optical power [1]. This feature suggests that the corresponding AWG modes, provided that their instability is

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slow enough, can be used in the presence of losses, as the losses would decrease the power without changing the propagation constant. This implies effective stability of the modes against dissipative degradation. Moreover, in the same situation the losses may additionally suppress the dynamical instability.

In this work, we focus on the study of asymmetric AWG modes produced by the above-mentioned spontaneous symmetry breaking at a bifurcation point. Note that asymmetric modes can also bifurcate from symmetric ones in another nonlinear symmetric planar structure, consisting of a linear core and nonlinear cladding [3]. However, they are different from the modes to be found in the present work. Moreover, the core and the cladding in that structure are made of different materials. The homogeneity of both the core thickness and refractive-index difference at the core-clad interface in this heterogeneous structure can be problematic from the technological viewpoint. In contrast, an AWG structure can be fabricated by means of diffusing an appropriate dopant, which alters the linear refractive index but does not conspicuously affect the Kerr coefficient, into a silica substrate. This fabrication mode has a great technological advantage.

The rest of the paper is organized as follows. The AWG model is formulated, and the asymmetric modes are investigated in it by means of analytical methods, in Sec. II. The instability of the modes is studied in Sec. III, the switching scheme based on the asymmetric modes and their instability is discussed in Sec. IV, and the results of the work are summarized in Sec. V.

II. ANALYSIS OF THE ANTIWAVEGUIDE MODES

We start with the standard NLS equation,

$$2ik\frac{\partial\Psi}{\partial\zeta}+\frac{\partial^2\Psi}{\partial\eta^2}+U(\eta)\Psi+|\Psi|^2\Psi=0, \tag{1}$$

where ζ and η are the propagation and transverse coordinates in the WG or AWG, $k=2\pi n/\lambda$, n is the linear refractive index, λ is the light wavelength, the nonlinearity coefficient is normalized to be 1, and

$$U(\eta)=\begin{cases}U_0, & |\eta|\leqslant\eta_c\\ 0, & |\eta|\geqslant\eta_c\end{cases} \tag{2}$$

(which corresponds to a step-index structure), $2\eta_c$ being the core thickness. For the WG and AWG, respectively, $U_0>0$ and $U_0<0$. To look for stationary AWG modes with a propagation constant β, we substitute into Eq. (1) $\Psi(\zeta,\eta)=\Phi(\zeta,\eta)\exp(i\beta\zeta)$, and rescale the equation according to

$$x\equiv\eta/\eta_c,\; z\equiv\zeta/(2k\eta_c^2),\; E\equiv\beta\eta_c^2,\; A\equiv U_0\eta_c^2,\; R=\Phi\eta_c\sqrt{2}, \tag{3}$$

which produces the basic propagation equation in a renormalized form,

$$i\frac{\partial R}{\partial z}+\frac{\partial^2 R}{\partial x^2}+[W(x)-E]R+2|R|^2R=0, \tag{4}$$

where $W(x)$ is the same potential as defined by Eq. (2) but with U_0 substituted by A [see Eq. (3)]. If $R(x)$ does not depend on z then $R(x)$ is a real function corresponding to the ground state of the linear Schrödinger equation in quantum mechanics, i.e., $R(x)$ has no zeros at finite values of x, and exponentially decays at $|x|\to\infty$. In this case, after rescaling and carrying out straightforward integration separately in the core and cladding, we obtain the equations

$$(dR/dx)^2=(E-A)R^2-R^4+C,\; |x|\leqslant 1, \tag{5}$$

$$(dR/dx)^2=ER^2-R^4,\; |x|\geqslant 1, \tag{6}$$

where C is an arbitrary integration constant, and A is positive for WGs and negative for AWGs. Here, a difference of the present problem from similar problems for the linear Schrödinger equation in quantum mechanics should be stressed. In the linear case, the energy parameter E takes discrete eigenvalues corresponding to the bound (localized) states. In the nonlinear case, the spectrum of the values of the (renormalized) propagation constant E pertaining to the localized states is *continuous*, because a new parameter comes into play, viz., the solution's amplitude, which, obviously, played no role in the linear case.

Note that Eq. (1) allows one to define a *diffraction length* z_D, which provides for a characteristic scale of the propagation distance (below, we will need it to compare with the distance over which a newly found mode remains effectively stable): $z_D\sim X^2$, X being the characteristic size of the beam in the transverse direction. In a typical case, X is about the core thickness, or, in terms of the rescaled variables defined in Eq. (3), we simply have $z_D\sim 2$.

The eigenfunctions determined by Eqs. (5) and (6) must be nonsingular solutions exponentially vanishing at $|x|\to\infty$. A straightforward consideration of the equations allows us to prove that, if the solution has a single extremum point inside the core, then the solution is symmetric and monotonically decaying as x varies between 0 and $\pm\infty$. Solutions with more than one extremum point in the core region are only possible for AWGs. Obviously, solutions with more than one extremum are nonmonotonic. Both symmetric and asymmetric nonmonotonic solutions are thus possible.

The symmetric AWG eigenmodes were considered in Ref. [4]. A characteristic feature of the eigenmodes in the AWG is their *multiplicity*, i.e., one may have more than one solution belonging to the same set of values (A,E). When the eigenmodes are multiple, they may indeed have several maxima and minima in the core. A typical example of a full set of symmetric and asymmetric AWG eigenmodes which have a single maximum in the core (but may also have minima) is shown in Fig. 1. Note that they all have no zeros at finite $|x|$.

The origin of the asymmetric modes can be easily understood. After simple manipulations, it follows from Eqs. (5) and (6) that the values of both $R^2(x)$ and $(dR/dx)^2$ must coincide at the two core-cladding interfaces, $x=\pm 1$, but the signs of dR/dx at these points may be opposite or equal, which gives rise, respectively, to the symmetric and asymmetric solutions. Accordingly, a *separatrix* between these two types of solutions is that with $dR/dx=0$ at $x=\pm 1$.

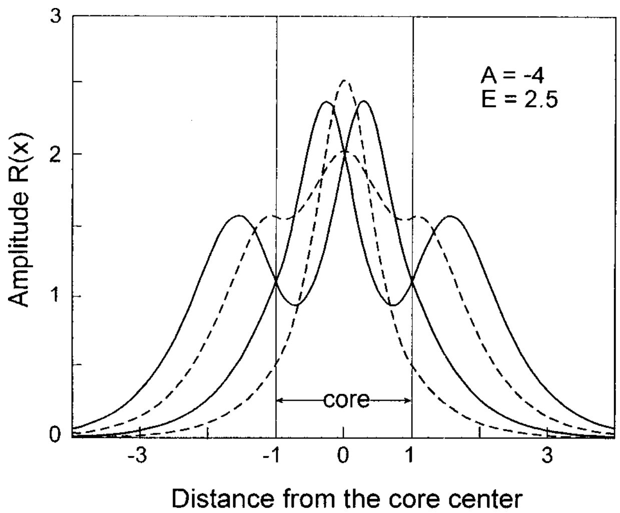

FIG. 1. An example of the full set of the antiwaveguide eigenmodes with one maximum in the core and no zeros, in the case $A=-4.0$, $E=2.5$. Here and in other figures normalized distance is defined by Eq. (3).

Asymmetric solutions with zeros at finite $|x|$ are also possible, but we do not consider them, as it seems likely that they are strongly unstable.

We now proceed to a detailed consideration of the asymmetric AWG solutions. The maximum and minimum values of the eigenfunction $R(x)$ inside the core, $R_{\max}$ and $R_{\min}$, are related as follows from Eq. (5):

$$R_{\min}^2+R_{\max}^2=E-A, \tag{7}$$

and Eq. (5) may be written as

$$\frac{dR}{dx}=\pm\sqrt{(R_{\max}^2-R^2)(R^2-R_{\min}^2)}. \tag{8}$$

Equation (8) can be solved in terms of incomplete elliptic integrals. However, for the actual analysis, it proves to be more convenient to define a variable φ and a parameter α as follows:

$$R^2\equiv R_{\max}^2\cos^2\varphi+R_{\min}^2\sin^2\varphi,\quad \sin^2\alpha\equiv(R_{\max}^2-R_{\min}^2)/R_{\max}^2, \tag{9}$$

and then obtain from Eq. (8)

$$I_1\equiv\int_0^{\pi/2}\sqrt{\frac{1+\cos^2\alpha}{1-\sin^2\alpha\sin^2\varphi}}d\varphi=\sqrt{E-A}. \tag{10}$$

Equation (11) defines the eigenvalues of α, and, accordingly, $R_{\max}$, as functions of $(E-A)$. It can be demonstrated that the integral I_1, as a function of α, attains a minimum value $\pi/\sqrt{2}$ at the point $\alpha=0$. Therefore, for the asymmetric modes, the parameter $(E-A)$ is bounded from below, $E-A\geqslant\pi^2/2$, and the equality in this relation is attained when $R_{\min}=R_{\max}=\pi/2$, i.e., on a degenerate mode with the constant value $R(x)\equiv\pi/2$ inside the core (it can be easily demonstrated that, in this case, $E=-A=\pi^2/4$).

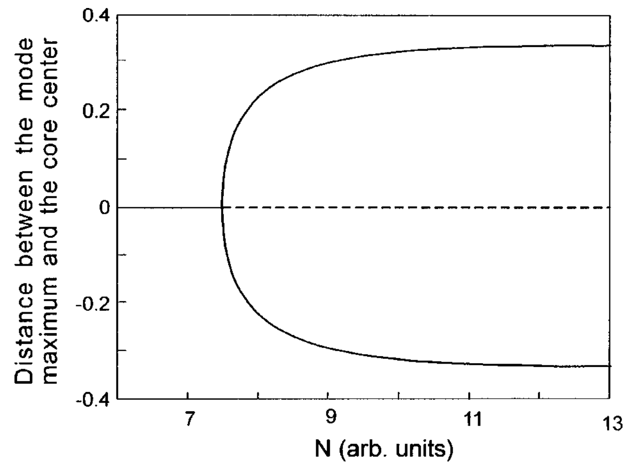

FIG. 2. A bifurcation diagram showing the formation of the asymmetric eigenmodes in the nonlinear antiwaveguide at $A=-4$.

The power of the WG or AWG mode is naturally defined as $N=\int_{-\infty}^{\infty}R^2dx$, which can be transformed into the expression

$$N=2\sqrt{E-A}I_2+2\sqrt{E},\quad I_2\equiv\int_0^{\pi/2}\sqrt{\frac{1-\sin^2\alpha\sin^2\phi}{1+\cos^2\alpha}}d\varphi. \tag{11}$$

Using the Cauchy-Bunyakovsky inequality $4I_1I_2\geqslant\pi^2$, we obtain a bound from below for the power of the AWG modes:

$$N\geqslant\pi^2/2+2\sqrt{E}. \tag{12}$$

The equality in Eq. (12) is attained for the above-mentioned degenerate mode with $R(x)\equiv\pi/2$ inside the core.

The transition from the symmetric to asymmetric AWG modes with increase of the intensity is, in fact, a typical example of a *transcritical pitchfork bifurcation* (see, e.g., [6]), as is illustrated by Fig. 2, showing a deviation of the eigenmode's maximum from the core center, Δx (which is a

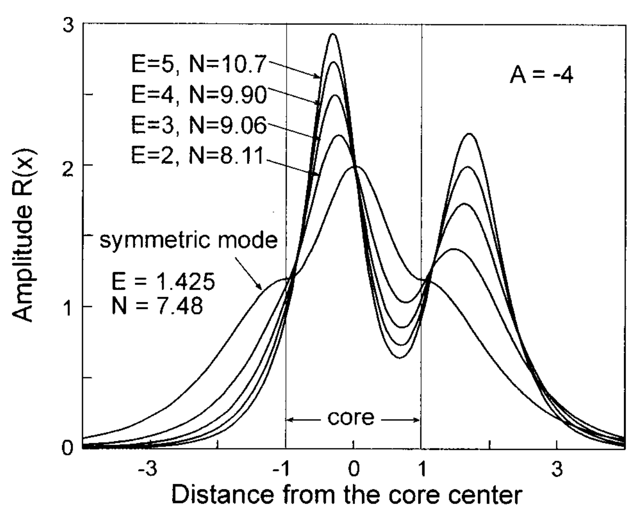

FIG. 3. An example of the evolution of the asymmetric antiwaveguide's eigenmode with increase of the beam's power N past the bifurcation point.

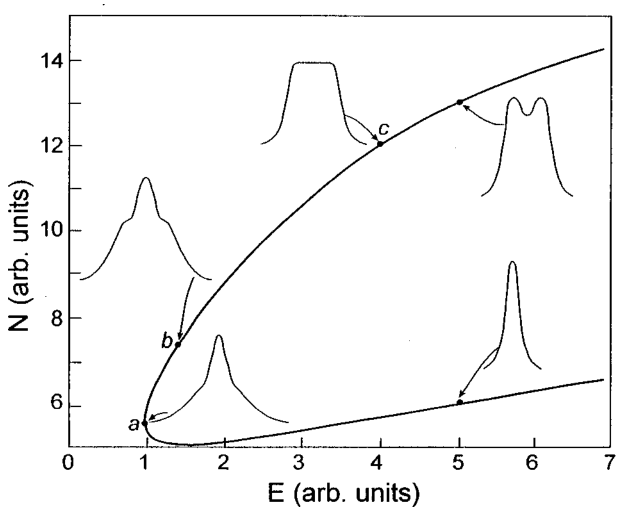

FIG. 4. An example of the dependence N vs E for the symmetric modes and evolution of these modes at $A=-4$; a is the tangent-bifurcation point, b the pitchfork-bifurcation point, and c the point corresponding to the degenerate solution with constant amplitude in the core.

natural measure of the solution's asymmetry), vs the power N, as obtained from the numerical solution. Note that spontaneous symmetry breaking described by a similar bifurcation diagram is well known in another nonlinear optical system, viz., a dual-core fiber [7].

A typical example of the evolution of the AWG eigenmode, starting from the bifurcation point, is displayed in detail in Fig. 3, where it is seen that the asymmetry may become very strong. Note that in the case illustrated by Fig. 3, the bifurcation starts from the simplest symmetric mode having a maximum at the center, $x=0$. A similar bifurcation starting from a more complicated symmetric mode which, instead, has a *minimum* at the center, has also been found (not shown here).

An example of the dependence N vs E for the symmetric mode and the evolution of this mode at $A=-4$ is shown in Fig. 4. The pitchfork bifurcation point b is not geometrically singled out on this curve. Note the presence of another, *tangent* bifurcation point a, at which two solution branches with opposite signs of the derivative dN/dE merge and disappear. Other conclusions clearly following from Fig. 4 are the existence of the above-mentioned finite threshold power for the formation of the symmetric mode, and the existence of a *gap*, in terms of the propagation constant E, for a given AWG potential-hill amplitude A.

III. STABILITY

A very important issue is stability of the AWG modes. A simple *necessary* stability condition is given by the *Vakhitov-Kolokolov* (VK) criterion [5], $dN/dE>0$ [obviously, Eqs. (10) and (11) define N as a function of E; see, e.g., Fig. 4]. Using the above results, it is possible to prove analytically, after lengthy transformations, that the asymmetric AWG mode, unlike the symmetric ones, *always* satisfies the VK criterion. Nevertheless, it does not seem feasible that any AWG-trapped mode may be fully stable. Indeed, the instability of the modes can be checked by means of a quite general Jones theorem [2], which states that a *sufficient instability* condition can be formulated in terms of an auxiliary Hermitian linear operator

$$\hat{L} \equiv \frac{d^2}{dx^2} + W(x) + 6R^2(x,E) - E \tag{13}$$

[which is written in terms of the notation defined by Eqs. (3)]: for instability of a given mode, it is sufficient that the operator $\hat{L}$ has, at least, two positive eigenvalues (casting the Jones' theorem into this form, simplified in comparison with the original formulation given in Ref. [2], we make use of the fact that we consider a mode with no zeros). The Jones theorem was earlier applied to show the instability of a symmetric model in the usual WG with a linear core and nonlinear cladding [8].

The positive eigenvalues of the operator can be found numerically, for a given mode $R(x)$. We have checked that, in all the cases considered, the Jones' criterion does predict an instability of the modes. For instance, in the case of the asymmetric mode with $A=-4$ and $E=4$, which turns out to be relatively weakly unstable in direct simulations (see Fig. 5), the positive eigenvalues of the operator $\hat{L}$ are 22.4, 12.0, and 1.20 for the solution without, with one, and with two zeros, respectively. The next solution with three zeros has negative eigenvalue -0.47. It may be relevant to mention here that in another nonlinear optical system where cw (continuous-wave) beams demonstrate formation of an asymmetric mode through a similar bifurcation, viz., a dual-core optical fiber [7], the asymmetric cw states, although they were tacitly assumed to be stable [7], were recently demonstrated to be strongly unstable [9].

The VK and Jones stability/instability criteria do not predict particular features of the instability, which are most important for applications, and must be studied by means of direct simulations. The simulations concur with the Jones theorem in showing that all the AWG modes are unstable. Typically (for moderate values of E), in the simulations the asymmetric modes persist over a propagation distance $\sim z_D$ (recall that z_D is the diffraction length, ~ 2 in the present notation) (see the example in Fig. 5 for $E=4$ and $A=-4$). Nevertheless, other simulations show that, with increase of E, the propagation distance before the onset of a conspicuous instability increases, and may become essentially larger then z_D. Even in the case when the AWG mode persists only over a propagation distance $\sim z_D$, in units of the core's half-width η_c it is, with regard to Eq. (3), $\zeta/(2k\eta_c^2) \sim z_D$. Usually, $\eta_c \sim \lambda$ and, as we said above, $z_D \sim 2$, hence a final estimate for the undisturbed propagation distance for the AWG mode is $\zeta \sim 33\mu$m (in the typical case $n \sim 1.5$, $\lambda \approx 1.5\mu$m) which is quite sufficient for switching applications (see below), and may be enough for direct experimental observation of the asymmetric mode.

The instability leads either to self-focusing of the beam in the core's center or to its expulsion into the cladding, similarly to what is shown in Fig. 6. In the latter case, the angle between the oblique beam propagation direction and the AWG axis is determined by the AWG parameters and the beam's power.

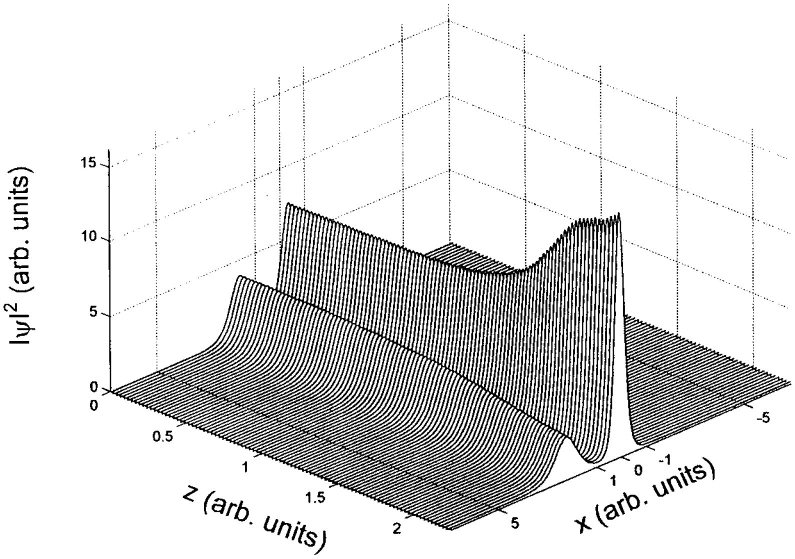

FIG. 5. A typical example of the numerically simulated evolution of the asymmetric mode along the propagation direction at $A=-4$, $E=4$. The input form of the mode is as in Fig. 3.

IV. APPLICATION OF SPONTANEOUS SYMMETRY BREAKING IN THE ANTIWAVEGUIDE TO SWITCHING

The bifurcation transforming the symmetric AWG mode into asymmetric ones (Fig. 2), in combination with the instability of the resultant asymmetric modes against walking into the cladding (the *deflecting instability*, see Fig. 6), can be used to implement all-optical switching. The AWG offers, in fact, the very convenient possibility of *controlling* the switching: launching a symmetric beam with a power exceeding (by not too much) the critical (bifurcation) value, the choice of one of the two mutually symmetric branches at the bifurcation point (see Fig. 2) can be determined by a ''hot spot,'' created off the AWG's center by a perpendicular laser beam focused on the waveguide surface, similar to what was proposed, in a different context, in Ref. [10]. The hot spot will attract the beam and break its symmetry via a local refractive-index change induced by the Kerr effect (see Fig. 6). In numerical simulations the hot spot is approximated by a small change of the refractive index $\sim 10^{-2}$–10^{-3} of the refractive-index difference between the core and cladding. In Fig. 6 the hot spot is shown having the same size in normalized units along the x and z axes. As it is noted above, in non-normalized units the z axis scale is 33 times greater than that of the x axis. However, the results do not differ considerably if the longitudinal length of the hot spot is decreased by one order of magnitude. The naturally enhanced sensitivity of the system to an external disturbance near the bifurcation point [6] makes it possible to considerably decrease the necessary power of the controlling beam that creates the hot spot. It is important to stress that, as is clearly seen in Fig. 6 (and in a number of other simulations performed at different values of the parameters), the deflecting instability of the beam does not give rise to any conspicuous fanning or stripping, keeping the coherent character of the beam. We stress that this finding is far from being trivial. For instance, in the

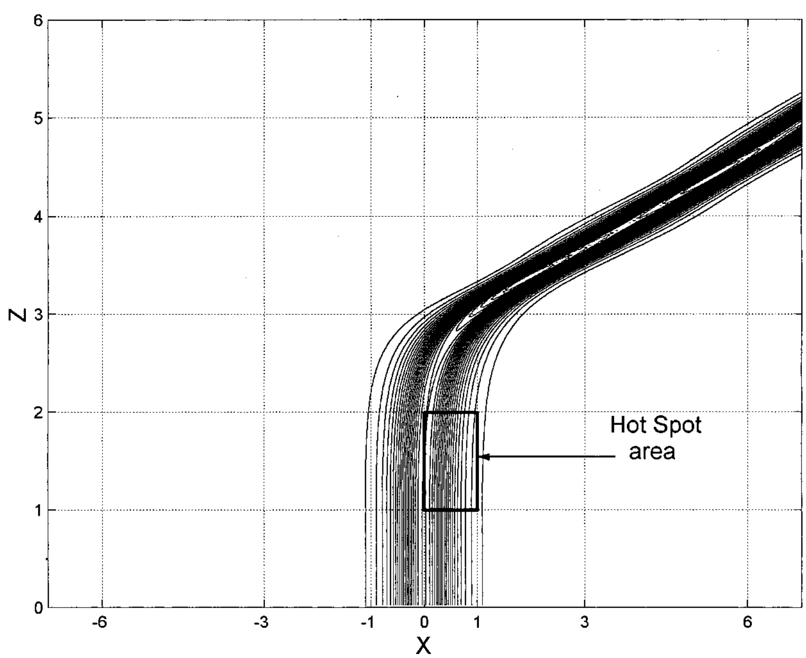

FIG. 6. An example of the controllable deflection of the guided beam initiated by the hot spot at $A=-4$, $E=4$.

multitrough switching model introduced in Ref. [10], where the switching was also controlled by means of a hot spot, a beam could be transferred from a given trough to an adjacent empty one, but this was accompanied by a strong disturbance (including stripping) of the beam.

As concerns the impact of the instability on the switching, note that there is a strategy for design of nonlinear switching devices assuming *stimulation* of the instability, when a short switching distance is required [11]. In the present case, the instability of the asymmetric mode that is generated by the bifurcation will help to complete the switching process in a shorter propagation distance ($\lesssim 33\mu$m, according to the estimate obtained above), driving the beam into the cladding. In fact, the same hot spot that helped to choose between the two asymmetric beams that might be generated by the original symmetric one can also easily provide for a disturbance that will stimulate the onset of the asymmetric-beam instability, pushing the beam into the cladding.

The single-channel configuration considered in this work can be extended to include several parallel antiwaveguides (cf. Refs. [10,12]), which might be a basis for multichannel devices. Promising applications of such devices include wavelength multiplexing, multichannel variable distribution or attenuation, time-domain multiplexing, etc. Moreover, even in the case of the single-input AWG channel, the scheme can be made multichannel of the $1 \rightarrow N$ type, by adding to it *several* output WG channels that will catch and trap the deflected beam, depending on the deflection angle (see Fig. 6). The latter angle, in turn, can be controlled by means of varying the intensity and position of the above-mentioned controlling hot spot. The extension of the AWG-based scheme in this direction demands detailed numerical calculations, which will be presented elsewhere [13].

Lastly, we note that, in this work, we considered the simplest step-index AWGs. However, one can check that essentially the same results are also true for graded-index AWGs.

V. CONCLUSION

In this work, we have found that an optical antiwaveguide induces a spontaneous transition from symmetric to asymmetric modes via a transcritical pitchfork bifurcation, provided that the beam's intensity exceeds a certain critical value, which is found numerically as a function of the antiwaveguide's strength. The asymmetric mode is subject to a mild deflecting instability, which drives it into the cladding, without giving rise to fanning or stripping of the beam. The bifurcation, in combination with the deflecting instability, may be used to design a device for all-optical switching. The switching can easily be controlled by means of a symmetry-breaking ''hot spot'' that acts upon an initial symmetric beam launched with a power exceeding the bifurcation value.

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